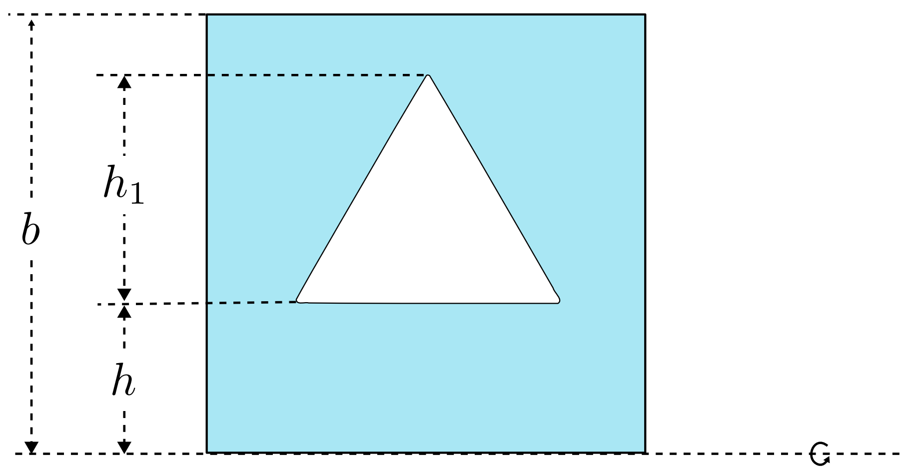
PHYS 232 – Assignment #4

Due Wednesday, Mar. 13 @ 11:00

1. The figure below shows a square of sides length b with a hole through it. The hole is in the shape of an equilateral triangle. The base of the triangle is a height h above the base of the square. The height of the triangle is h_1.



A student has measured $b = 15.4 \pm 0.6$ cm and $h_1 = 8.6 \pm 0.4$ cm. Find the area of the shape shown in the figure and its uncertainty. Give your answer in units of cm^2.

BONUS: Imagine that the square is now rotated 360° about an axis that passes through its base (the long dashed line). The resulting shape is a cylinder with a cavity. The radius and length of the cylinder are both b. The cavity is a torus (doughnut) with a cross-section of the equilateral triangle.

If $h = 3.3 \pm 0.2$ cm, find the volume of the 3-D shape swept out by the rotation (i.e. the volume of the cylinder minus the volume of the torus with triangular cross-section) and its uncertainty in units of cm^3. To receive bonus marks, you must clearly explain all steps used to calculate the volume. No credit will be given for simply writing down the correct answer or if important steps are skipped or not clearly explained.

2. (a) Two resistors R_1 and R_2 are connected in series such that the equivalent resistance is $R = R_1 + R_2$.

(i) Write down an expression for $(\Delta R)^2$ in terms of ΔR_1 and ΔR_2.

(ii) Now assume that R_1 and R_2 have the same percent uncertainty such that $\Delta R_1 = xR_1$ and $\Delta R_2 = xR_2$ where $0 < x < 1$. Re-express $(\Delta R)^2$ in terms of x, R_1, and R_2.

(iii) Now determine the values of R_1 and R_2 (in terms of R) that minimize $(\Delta R)^2$.
Hint: Start by making the substitution $R_2 = R - R_1$.

(iv) Finally, write down an expression for ΔR in terms of x and R. Assume that R_1 and R_2 have the values calculated in part (iii). If R_1 and R_2 are 5% resistors, what is the percent uncertainty in R?

(b) Two resistors R_1 and R_2 are connected in parallel such that the equivalent *conductance* is $G = R_1^{-1} + R_2^{-1}$ where $G \equiv R^{-1}$.

(i) Write down an expression for $(\Delta G)^2$ in terms of R_1, R_2, ΔR_1 and ΔR_2.

(ii) Now assume that R_1 and R_2 have the same percent uncertainty such that $\Delta R_1 = xR_1$ and $\Delta R_2 = xR_2$ where $0 < x < 1$. Re-express $(\Delta G)^2$ in terms of x, R_1, and R_2.

(iii) Now determine the values of R_1 and R_2 (in terms of R) that minimize $(\Delta G)^2$.
Hint: Start by making the substitution $\dfrac{1}{R_2} = \dfrac{1}{R} - \dfrac{1}{R_1}$.

(iv) Finally, write down an expression for ΔR in terms of x and R. Assume that R_1 and R_2 have the values calculated in part (iii). If R_1 and R_2 are 5% resistors, what is the percent uncertainty in R? Remember that $R = 1/G$.

3. According to Bohr's model of the atom, the energy of the electronic quantum states of Hydrogen are given by:

$$E_n = -\left(\frac{me^4}{8\varepsilon_0^2 h^2}\right)\frac{1}{n^2}$$

where m is the electron mass, e is the fundamental charge, ε_0 is the permittivity of free space, h is Planck's constant, and $n = 1, 2, 3, \ldots$ is the principal quantum number. Suppose the uncertainty in ε_0 is negligible and the relative uncertainties in m, e, and h are 0.005, 0.002, and 0.01, respectively. That means, for example, that $\Delta m/m = 0.005$. Find the uncertainty in the energy of the $n = 4$ state of the hydrogen atom. Give your answer in units of Joules.

4. The Sackur-Tetrode equation is an expression for the entropy of a monatomic ideal gas:

$$S = k_\mathrm{B} N \left\{ \ln\left[\frac{N}{V}\left(\frac{4\pi m U}{3Nh^2}\right)^{3/2}\right] + \frac{5}{2}\right\}.$$

Here, N is the number of particles, V is the volume of the gas, m is the mass of a single gas particle, h is Planck's constant, and $U = \frac{3}{2}Nk_\mathrm{B}T$ is the internal energy of the gas.

Assume that the uncertainties in k_B and h are negligible and that $N \pm \Delta N$, $V \pm \Delta V$, $m \pm \Delta m$, and $T \pm \Delta T$ have all been measured. Derive an expression for ΔS, the uncertainty in the entropy.